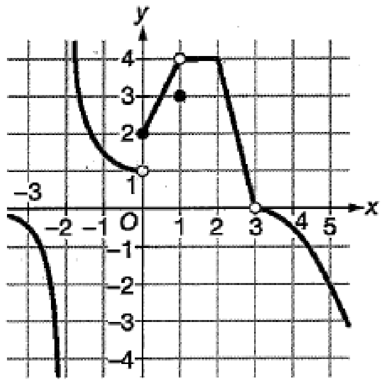

Graph of f

x	0.8	0.9	0.09	0.009	1.001	1.01	1.1	1.2
$g(x)$	4.16	4.59	4.960	4.996	5.004	5.040	5.39	5.76

1) The graph of the function f is shown in the xy-plane above. The graph of f has a vertical asymptote at $x=-2$. The function g is continuous and increasing for all x. Values of $g(x)$ at selected values of x are shown in the table above.

a) Using the graph of f and the table for g, estimate $\lim\limits_{x\to 1}\left(2f(x)+3g(x)\right)$.

$$\underbrace{2\lim_{x\to 1} f(x)}_{8} + \underbrace{3\lim_{x\to 1} g(x)}_{3(5)} = 23$$

b) For each of the values $a=-2$, $a=2$, and $a=3$, determine whether or not f is continuous at $x=a$. In each case, the three-part definition of continuity to justify your answer.

$a=-2$

$f(x)$ is not continuous at $x=-2$ b/c $f(-2)=\emptyset$.

$a=2$

$f(2)=4$

$\lim\limits_{x\to 2} f(x)=4$

$f(2)=\lim\limits_{x\to 2} f(x)$

$a=3$

$f(3)=\emptyset$

c) Find the value of $\lim\limits_{x\to 0} f\left(f(x)\right)$ or explain why the limit does not exist.

$$\left.\begin{aligned}\lim_{x\to 0^-} f(f(x)) &= \lim_{x\to 1^+} f(x) = 4\\ \lim_{x\to 0^+} f(f(x)) &= \lim_{x\to 2^+} f(x) = 4\end{aligned}\right\} \lim_{x\to 0} f(f(x)) = 4$$

2) The function, $Y(t)$, is a piecewise-defined function defined by:

$$Y(t)=\begin{cases} 10e^{0.05t} & \text{for } 0 \le t \le 10 \\ f(t) & \text{for } 10 < t \le 12 \\ \dfrac{600}{20+10e^{-0.05(t-12)}} & \text{for } t > 12 \end{cases},$$

where $f(t)$ is a continuous function such that $f(12)=20$.

$$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

a) Find $\lim_{t\to\infty} Y(t)$.

$$\lim_{t\to\infty} \frac{600}{20+10e^{-0.05(t-12)}} = \frac{600}{20+10e^{-\infty}} = \frac{600}{20} = 30$$

b) Is the function $Y(t)$ continuous at $t=12$. Justify your answer.

I. $Y(12)=f(12)=20$

II. $\left.\begin{array}{l}\lim_{t\to 12^-} Y(t)=20 \\ \lim_{t\to 12^+} Y(t)=20\end{array}\right\} \lim_{t\to 12} Y(t)=20$

III $Y(12)=\lim_{t\to 12} Y(t)$

c) The function Y is continuous at $t=10$. Is there a time t, for $0<t<12$, at which $Y(t)=18$. Justify your answer.

Since $\left\{\begin{array}{l} Y(0)=10 < 18 \\ Y(12)=20 > 18 \end{array}\right\}$ & $Y(t)$ is continuous

DATE	CONCEPT	IN-CLASS SAMPLE PROBLEMS
	CONTINUITY	**Types of Discontinuities** **Ex. 1** What type of discontinuity does $f(x)=\frac{1}{x}$ have at $x=0$? Non-removable (Infinite) **Ex. 2** What type of discontinuity does $f(x)=\frac{x-3}{x^2-9}$ have at $x=3$? At $x=-3$? Removable @ $x=3$ Infinite @ $x=-3$ **Removing a Discontinuity** **Ex. 1** Define $g(5)$ in a way that extends $g(x)=\frac{x^2-25}{x-5}$ to be continuous at $x=5$ $g(x)=\frac{x^2-25}{x-5} \rightarrow \frac{(x+5)(x-5)}{x-5}$ For g to be cont. $g(5)=10$ removable disc. @ $x=1$ **Ex. 2** Let $f(x)=\begin{cases}\frac{x^2-3x+2}{x^2-4x+3}, & \text{when } x\neq 1\\ k, & \text{when } x=1\end{cases}$. Find k to make $f(x)$ continuous. $k=\lim_{x\to 1}\frac{x^2-3x+2}{x^2-4x+3} \Rightarrow \lim_{x\to 1}\frac{(x-1)(x-2)}{(x-1)(x-3)} \rightarrow \frac{1}{2}$ $k=\frac{1}{2}$ removable disc @ $x=4$ **Ex. 3** Let $f(x)=\begin{cases}\frac{x^2-16}{\sqrt{x}-2}, & \text{when } x\neq 4\\ w, & \text{when } x=4\end{cases}$. Find w to make $f(x)$ continuous. $\lim_{x\to 4}\frac{x^2-16}{\sqrt{x}-2}$ $\lim_{x\to 4}\frac{(x-4)(x+4)}{\sqrt{x}-2}$ $\lim_{x\to 4}\frac{(\sqrt{x}+2)(\sqrt{x}-2)(x+4)}{(\sqrt{x}-2)}=32$ $w=32$
HOMEWORK		Worksheet 4